Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 1	DAILY TUTORIAL SHEET 3

51.(A) ASSASSINATION has AAA, SSSS, II, NN, T, O

(a) All alike $^{1}C_1$ **(b)** 3 alike, 1 different $^{2}C_1\,^{5}C_1$

(c) 2 alike, 2 alike $^{4}C_2$ **(d)** 2 alike, 2 different $^{4}C_1\,^{5}C_2$

(e) All different $^{6}C_4$ $\Rightarrow$ Total number of ways to select $= 1+10+6+40+15 = 72$

52.(A) We have A, A, I, I, N, N, E, M, O, T, X

Case 1: 2 alike of one kind, 2 alike of second kind. Number of words $= {}^{3}C_2 = 3$

Case 2: 2 alike of one kind, 2 different $^{3}C_1 \times {}^{7}C_2 = 63$

Case 3: All different $^{8}C_4 = 1670$

53.(A) 2 red, 2 green, 2 white strips

(a) 3 alike (Not possible) **(b)** 2 alike, 1diff. $^{3}C_1\,^{2}C_1 \times \frac{3!}{2!} = 18$

(c) All diff. $^{3}C_3 \times 3! = 6$ $\Rightarrow$ Total number of flags = 18 + 6 = 24

54.(A) 2 apples, 3 oranges, 4 Bananas

(a) 3 bananas $\equiv$ 1 way **(b)** 2 bananas, 1 apple/orange $\equiv$ 2 ways

(c) 1 banana, 2 apples $\equiv$ 1 **(d)** 1 banana, 2 oranges $\equiv$ 1 way

(e) 1 banana, 1 apple, 1 orange $\equiv$ 1 way $\Rightarrow$ Total ways = 6

55.(C) V, Y, M, N, R, II, DD, AA

(a) One I, 1 alike, 1 diff. $^{2}C_1\,^{6}C_1 \times \frac{4!}{2!} = 144$ **(b)** one I, 3 diff. $^{7}C_3 \times 4! = 840$

(c) Two I, 2 alike $^{2}C_1 \times \frac{4!}{2!\,2!} = 12$ **(d)** Two I, 2 diff. $^{7}C_2 \times \frac{4!}{2!} = 252$

Total ways $= 144+840+12+252 = 1248$

56.(C)

S. No.	P-1	P-2	P-3
1	3	1	1
2	2	2	1

Case 1: 3 perfumes of one variety and 1 each of other two varieties.

Number of ways $= \frac{5!}{3!\,1!\,1!} \times \frac{3!}{2!} = 60$

Case II: 1 perfume of one variety ad 2 each of other two varieties.

Number of ways $= \frac{5!}{2!2!1!} \times \frac{3!}{2!} = 90$

Total number of ways $= 60+90 = 150$

57.(B) $(AA....p)(BB....P).....(NN.....p)$

$(p+1)(p+1).....$ n times $= (p+1)^n$

But subtract 1 for the case when no book is selected. Hence number of ways $= (p+1)^n - 1$

58.(C) $(aa.......m)[b.c....n$ items$]$

Number of factors $= (m+1)2^n$

Subtract 1 for the case when no number is selected. Hence number of ways $= (m+1)2^n - 1$

59.(D) We have: $2160 = 2^4 \times 3^3 \times 5^1$
The total number of divisors is same as the number ways of selecting some or all out of four 2's, three 3's and one 5. The number of such ways is (4 + 1) (3 + 1) (1 + 1) – 1 = 39.

60.(D) For each book, we have (p + 1) choices. As there are n different books, number of ways of selecting books $= (p+1) \times (p+1) \times \ldots .n$ times $= (p+1)^n$.
But when we choose 0 copies of each book, we don't choose any book at all.
Number of ways of selecting 1 book is n. $\Rightarrow$ The required number of ways $= (p+1)^n - n$.

61.(B) Use: ${}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n} = 2^{2n} - 1$

62.(A) $216 = 2^3 \times 3^3$
Odd divisors have powers of 3 only $\Rightarrow$ Number of odd divisors = 4

63.(B) $3^p 6^m .21^n = 3^p .2^m .3^n .7^n = 2^m .3^{p+m+n} .7^n$
Number of odd divisors $= (p+m+n+1)(n+1) - 1$

64.(B) $1 \le m < n \le p$
$A = \{1, 2, 3, \ldots\ldots\ldots, p\}$ And $B = (m, m+1, \ldots\ldots\ldots, n-1, n)$
No. of elements $= n - m + 1$
As m and n will always be there in every subset, no. of elements in consideration are $(n-m+1) - 2 = n - m - 1$.
No. of subsets $= 2^{n-m-1}$

65.(A) $38808 = 2^3 .3^2 .7^2 .11$
No, of divisors $= (3+1).(2+1).(2+1).(1+1) = 4 \times 3 \times 3 \times 2 = 72$
But it includes 1 and the number itself
Hence, required no if divisors $= 72 - 2 = 70$

66.(A) *SURITI*

- *I* - - - - 5! words
- *R* - - - - 5!/2! words
- *S*
  - *SI* - - - - 4! words
  - *SR* - - - - 4!/2! words
  - *ST* - - - - 4!/2! words
  - *SU*
    - *SUI* - - - 3! words
    - *SURI*
      - *SURII* $\to$ 1
      - *SURITI* $\to$ 1

Rank of SURITI in the dictionary $= 5! + \frac{5!}{2!} + 4! + \frac{4!}{2!} + \frac{4!}{2!} + 3! + 1 + 1 = 236$

67.(C) Letters appearing in the word SACHIN are A, C, H, I, N, S
The words beginning with letters A, C, H, I and N appear before the word SACHIN.
There are 5(5!) = 600 words beginning with A, C, H, I and N
Word SACHIN is the first word beginning with S. Therefore, SACHIN appears at serial number 601.

68.(C) Letters appearing in the word COCHIN are C, C, H, I, N, O
Words appearing before COCHIN are of the form C X . . . . .
Where X is one of the letters C, H, I, N and the four remaining places can be filled by the remaining four letters.
Thus, the number of words before COCHIN is (4) (4!) = 96

69.(A) In dictionary the words at each stage are arranged in alphabetical order. Starting with the letter A, and arranging the other four letters GAIN, we obtain 4! = 24 words.

Thus, there are 24 words which start with A. These are the first 24 words.
Then, starting with G, and arranging the other four letters A, A, I, N in different ways, we obtain $\frac{4!}{2!} = \frac{24}{2} = 12$ words.

Thus, there are 12 words, which start with G.

Now, we start with I. The remaining 4 letters A, G, A, N can be arranged in $\frac{4!}{2!} = 12$ ways. So, there are 12 words, which start with I. Thus, we have so far constructed 48 words. The 49th word is NAAGI and hence the 50th word is NAAIG.

70.(B) Answer = ${}^{m}C_2 \,.\, {}^{n}C_2$

71.(D) The number of triangles = Total number of triangles – No. of triangles having one side common with the octagon – No. of triangles having two sides common with the octagon $= {}^{8}C_3 - {}^{8}C_1 \times {}^{4}C_1 - 8 = 16$.

72.(A) The number of points in which the lines can intersect = the number of ways of choosing two lines out of 25

$${}^{25}C_2 = \frac{25 \times 24}{2} = 300$$

73.(A) Line "1" will intersect the other lines in (n-1) points.
Line "2" will intersect the other lines in (n-2) points and so on.

Therefore, total number of different points of intersection $= \sum_{k=1}^{n-1} k = \frac{n(n-1)}{2}$

74.(B) If no three points were collinear, then the total number of quadrilaterals $= {}^{n}C_4$

Out of which ${}^{m}C_3 \left({}^{n-m}C_1\right) + {}^{m}C_4$ are not quadrilaterals.

75.(B) Given, $T_n = {}^{n}C_3$

$$T_{n+1} = {}^{n+1}C_3$$